Manthan: A Data-Driven Approach for Boolean Functional Synthesis

Priyanka Golia[1,2]

Joint work with Kuldeep S. Meel[1] and Subhajit Roy[2]

[1]National University of Singapore
[2]Indian Institute of Technology, Kanpur

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Boolean Functional Synthesis

- Given: A Boolean relation $F(X, Y)$, with inputs $X = \{x_1, \ldots, x_n\}$, and outputs $Y = \{y_1, \ldots, y_m\}$

- Problem: Synthesise a function vector $\Psi = \langle \psi_1 \ldots \psi_m \rangle$, where ψ_i is a function for variable y_i, $y_i = \psi_i(x_1, \ldots, x_n)$ such that

$$\exists Y F(X, Y) \equiv F(X, \Psi(X))$$

- Each ψ_i is called Skolem function, and Ψ is called Skolem function vector.

Skolem Function: Example

Let $X = \{x_1, x_2\}$ and $Y = \{y_1\}$

$F(X, Y) : x_1 \vee x_2 \vee y_1$

Skolem functions:

$$x_1 \vee x_2 \vee y_1 \longrightarrow \neg(x_1 \vee x_2) \rightarrow y_1$$

$$\downarrow$$

$$\psi_1(x_1, x_2) = \neg(x_1 \vee x_2)$$

$F(X, \Psi(X)) = x_1 \vee x_2 \vee \neg(x_1 \vee x_2)$

Skolem Function: Example

Let $X = \{x_1, x_2\}$ and $Y = \{y_1\}$ and $F(x_1, x_2, y_1) : \; x_1 \vee x_2 \vee y_1$

Possible Skolem function: $\psi_1(X) = \neg(x_1 \vee x_2)$

$$F(X, \Psi(X)) = x_1 \vee x_2 \vee \neg(x_1 \vee x_2)$$

X	$\exists Y F(X, Y)$		$F(X, \Psi(X))$
$x_1 = 0 \, , x_2 = 0$	$y_1 = 1$	True	True
$x_1 = 0 \, , x_2 = 1$	$y_1 = 1$	True	True
$x_1 = 1 \, , x_2 = 0$	$y_1 = 1$	True	True
$x_1 = 1 \, , x_2 = 1$	$y_1 = 1$	True	True

$\exists Y F(X, Y) \equiv F(X, \Psi(X))$

Skolem Function: Example

- $F(x_1, x_2, y_1) : \ x_1 \vee x_2 \vee y_1$
- Possible Skolem functions:

$$\psi_1(x_1, x_2) = \neg(x_1 \vee x_2)$$

$$\psi_1(x_1, x_2) = \neg x_1$$

$$\psi_1(x_1, x_2) = \neg x_2$$

$$\psi_1(x_1, x_2) = 1$$

Applications

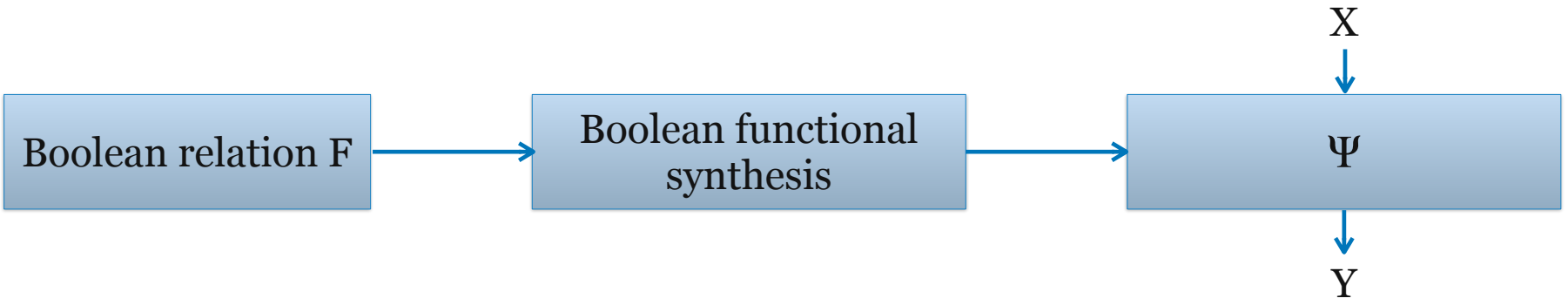
- Application in a wide variety of domains:
    - Certified QBF solving
    - Program synthesis
    - Cryptography

Factorization Problem

- Given a n-bit number X, find m-bit number Y such that Y divides X and $Y \notin \{1, X\}$.

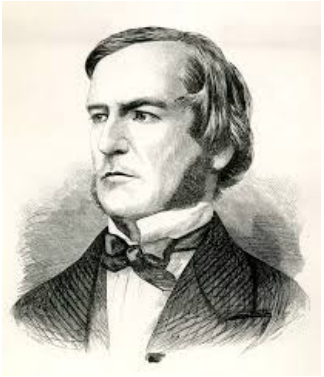
- As a relation between input and output values

$$F(X, Y) : \frac{X}{Y} \in \mathbb{Z} \text{ and } Y \notin \{1, X\}$$



$$\exists Y F(X, Y) \equiv F(X, \Psi(X))$$

A Long History



George Boole

Thoralf Albert Skolem

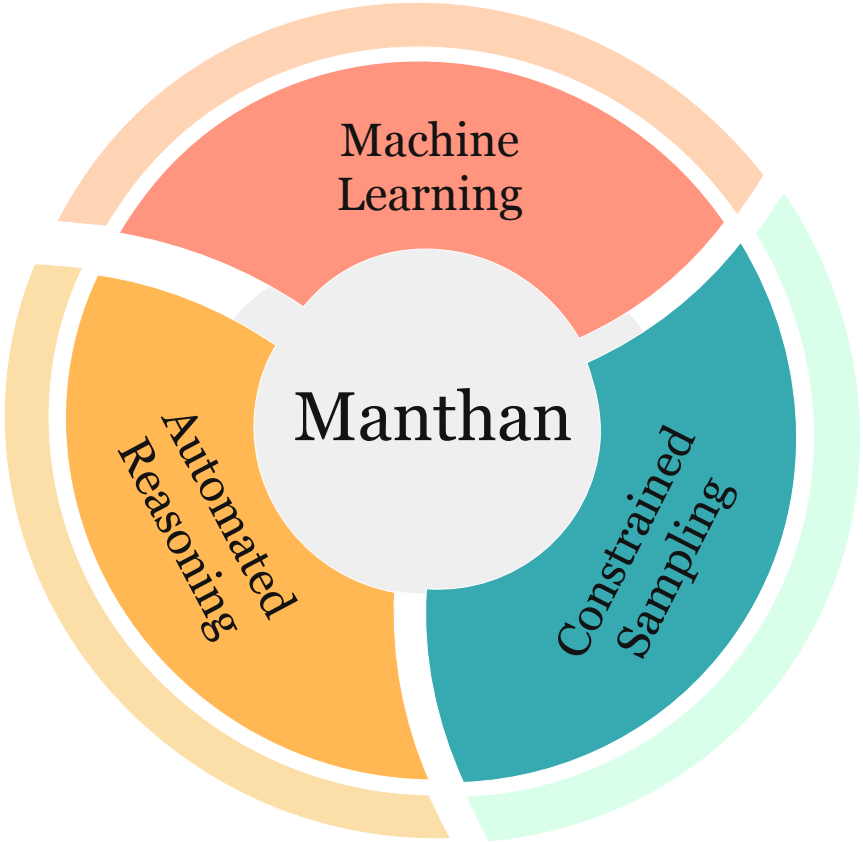
- The Customary Bad News

  Unless P = NP, there exist problem instances where Boolean function synthesis must take super-polynomial time. (Akshay *et al.* ,2018)

And Now, the Real Deal

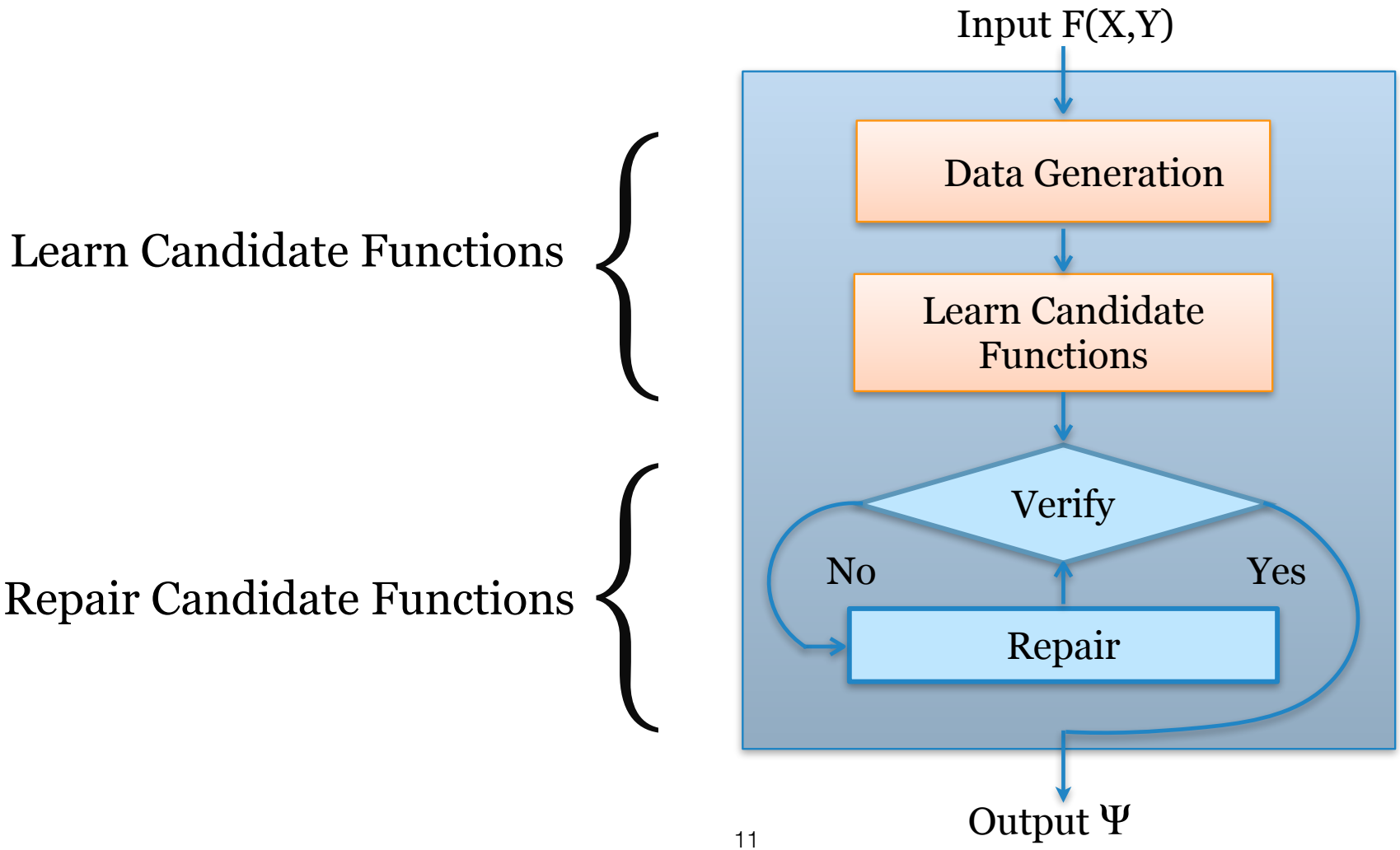
- From the proof of validity of $\forall X \exists Y F(X, Y)$
  - Bendetti et al., 2005
  - Jussilla et al., 2007
  - Heule et al., 2014
- Quantifier instantiation in SMT solvers
  - Barrett et al., 2015
  - Bierre et al.,2017
- Input-Output separation:
  - Chakraborty et al., 2018
- Knowledge representation:
  - Kukula et al., 2000
  - Trivedi et al., 2003
  - Jiang, 2009
  - Kuncak et al., 2010
  - Balabanov and Jiang, 2011
  - John et al., 2015
  - Fried, Tabajara, Vardi, 2016,2017
  - Akshay et al., 2017,2018
  - Chakraborty et al., 2019
- Incremental determinization:
  - Rabe et al., 2015, 2018, 2019

A Data-Driven Approach for Boolean Functional Synthesis



The Name, **Manthan**, is based on an Indian Mythological Story

Manthan


Input F(X,Y)
Learn Candidate Functions
Repair Candidate Functions
Data Generation
Learn Candidate Functions
Verify
No
Yes
Repair
Output Ψ

Data Generation

- We want to capture the relation between X and Y.

$F(x_1, x_2, y_1, y_2)$: $(x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

Unlike classical machine leaning
for the same valuation of x_1, x_2 : different y_1 , y_2

Data Generation

- $F(x_1, x_2, y_1, y_2)$: $(x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

Sample 4 data points uniformly at random

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0

- Possible Skolem function

$\psi_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad \psi_1(x_1, x_2) = \neg x_1$ $\quad \psi_1(x_1, x_2) = \neg x_2$ $\quad \psi_1(x_1, x_2) = 1$

$\psi_2(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad \psi_2(x_1, x_2) = \neg x_1$ $\quad \psi_2(x_1, x_2) = \neg x_2$ $\quad \psi_2(x_1, x_2) = 0$

Data Generation

- $F(x_1, x_2, y_1, y_2)$: $(x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

The magical sampler
4 data points →

x_1	x_2	y_1	y_2
0	0	1	0
0	1	1	0
1	0	1	0
1	1	1	0

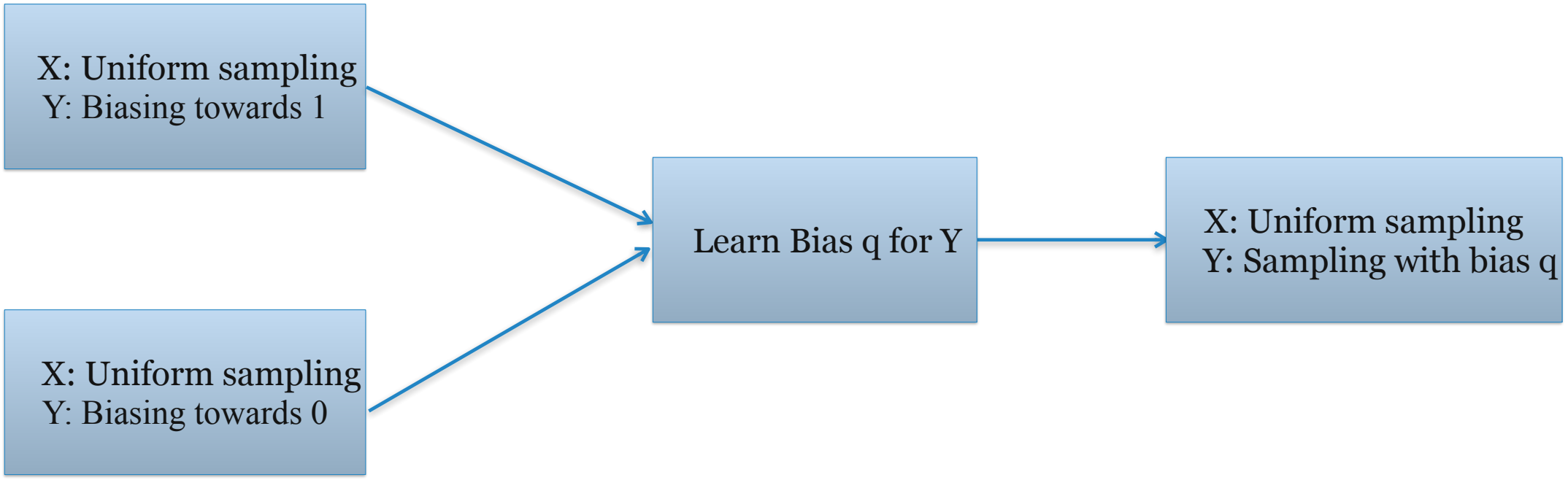
- Possible Skolem function

$\psi_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad \psi_1(x_1, x_2) = \neg x_1$ $\quad \psi_1(x_1, x_2) = \neg x_2$ $\quad \psi_1(x_1, x_2) = 1$

$\psi_2(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad \psi_2(x_1, x_2) = \neg x_1$ $\quad \psi_2(x_1, x_2) = \neg x_2$ $\quad \psi_2(x_1, x_2) = 0$

Data Generation

- We design a weighted sampling strategy that seeks to uniformly sample X, while biasing the valuation of Y.

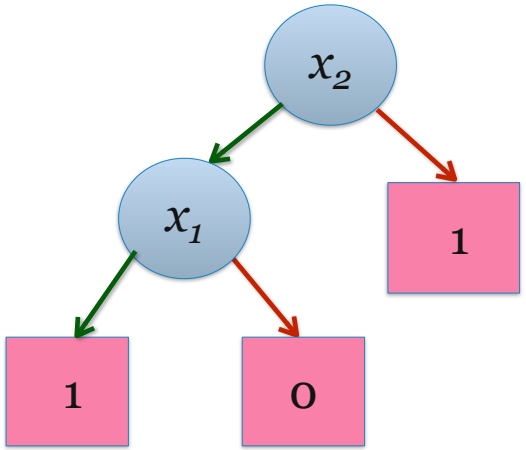


Learning Candidate Functions

- $F(x_1, x_2, y_1)$: $x_1 \vee x_2 \vee y_1$
  - Feature set: valuation of x_1, x_2
  - Label: valuation of y_1
  - Learn decision tree to represent y_1 in terms of x_1, x_2.

x_1	x_2	y_1
0	0	1
0	1	1
1	0	0
1	1	1

Binary classification problem.



$$\psi_1 = x_2 \vee (\neg x_2 \wedge \neg x_1)$$

Learning Candidate Functions

- To learn ψ_i :
  - Feature set: valuation of X in data
  - Label: valuation of y_i in data
  - Learn decision tree classifier
  - Candidate function ψ_i is disjunction of all the paths of the tree with leaf node 1

Verification of Candidate Functions

- When Candidate functions are not Skolem functions:

$$\exists Y F(X, Y) \not\equiv F(X, \Psi(X))$$

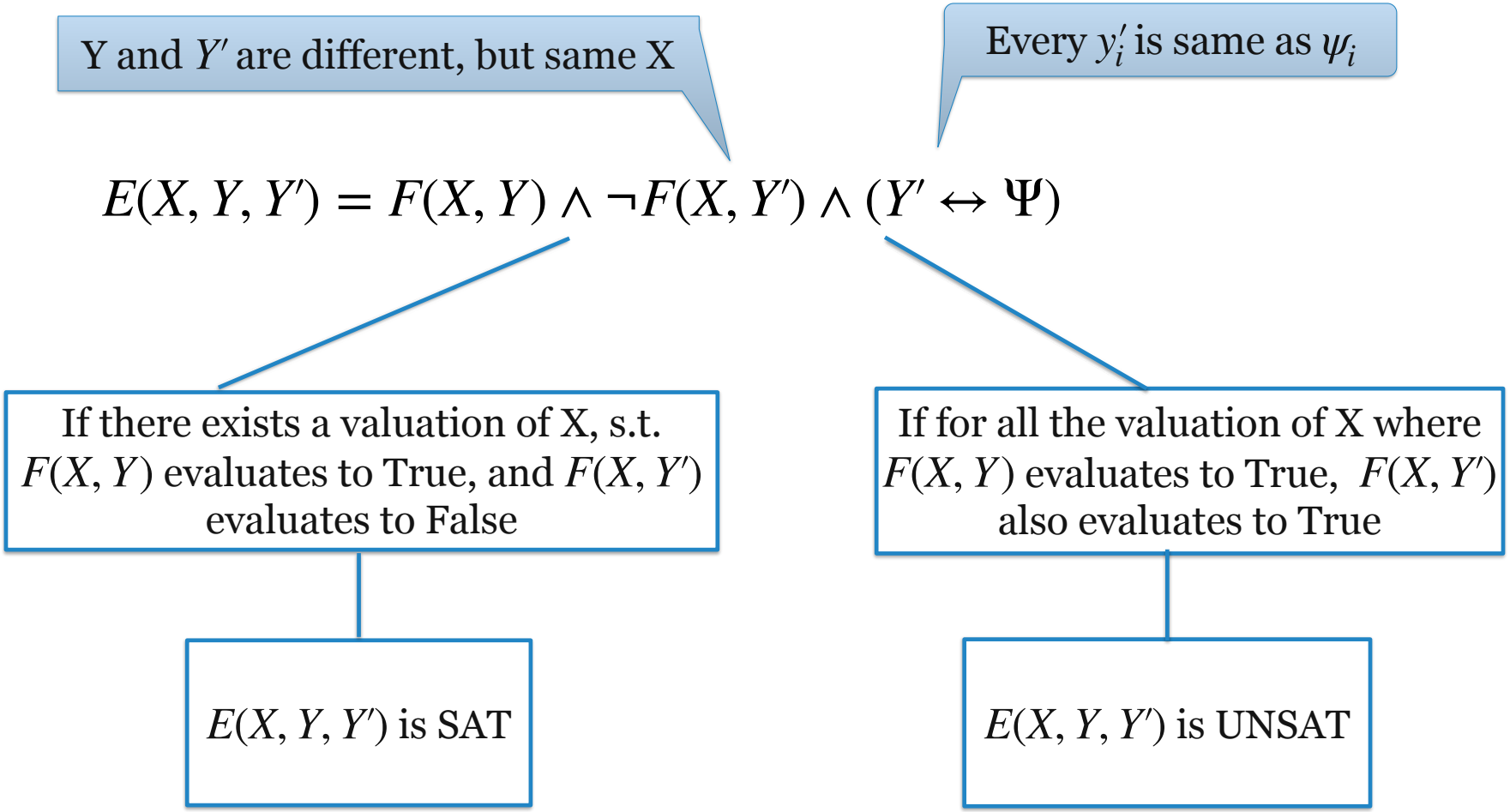
There exists at least one valuation of X where $\exists Y F(X, Y)$ evaluates to True, and $F(X, \Psi(X))$ also evaluates to False.

- When Candidate functions are Skolem functions:

$$\exists Y F(X, Y) \equiv F(X, \Psi(X))$$

For all the valuation of X where $\exists Y F(X, Y)$ evaluates to True, $F(X, \Psi(X))$ also evaluates to True.

Verification of Candidate Functions



Verification of Candidate Functions

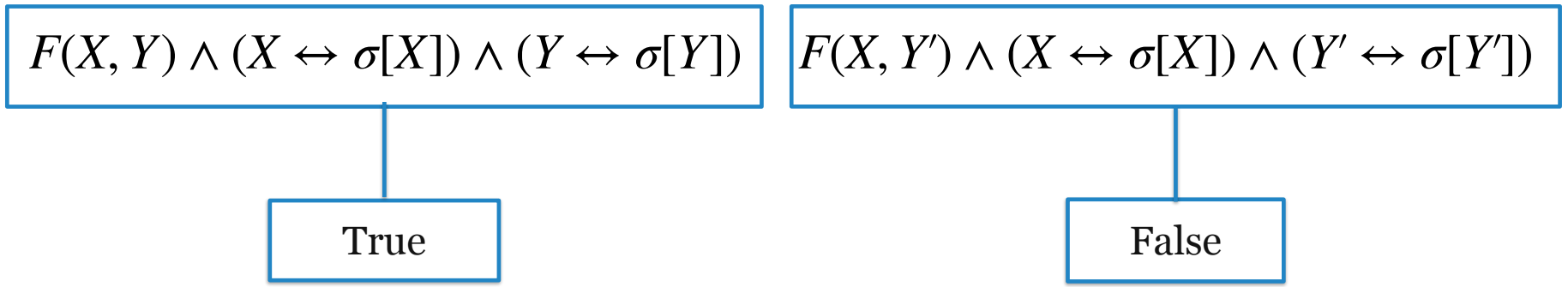
$$E(X, Y, Y') = F(X, Y) \wedge \neg F(X, Y') \wedge (Y' \leftrightarrow \Psi)$$

- Check satisfiability of $E(X, Y, Y')$.
- If $E(X, Y, Y')$ is UNSAT: return the Skolem function Ψ.
- If $E(X, Y, Y')$ is SAT: let $\sigma \vDash E(X, Y, Y')$ be a counter example to fix.

Repairing Candidate Functions

$$E(X, Y, Y') = F(X, Y) \wedge \neg F(X, Y') \wedge (Y' \leftrightarrow \Psi)$$

- $E(X, Y, Y')$ is SAT: $\sigma \models E(X, Y, Y')$



- The potential candidates to repair : functions corresponding to y_i, if $\sigma[y_i] \neq \sigma[y_i']$

Repairing Candidate Functions

- The aim of a repair iteration is to make $F(X, Y') \wedge (Y' \leftrightarrow \Psi)$ evaluates to True with $(X \leftrightarrow \sigma[X])$.

- Let candidate function $\psi_i(X)$ corresponding to y_i needs to repair.

  With $X \leftrightarrow \sigma[X]$

Before repair	After repair
$\psi_i(X) \mapsto 0$	$\psi_i(X) \mapsto 1$
$\psi_i(X) \mapsto 1$	$\psi_i(X) \mapsto 0$

Repairing Candidate Functions

- Construct a repair formula β (a subset of literals in $\sigma[X]$)

Before repair	Repair	After repair:
$\psi_i(X) \mapsto 0$	$\psi_i(X) \leftarrow \psi_i(X) \vee \beta$	$\psi_i(X) \mapsto 1$
$\psi_i(X) \mapsto 1$	$\psi_i(X) \leftarrow \psi_i(X) \wedge \neg\beta$	$\psi_i(X) \mapsto 0$

Repairing Candidate Functions

- Find UNSAT core of $G_i(X, Y) = F(X, Y) \wedge (X \leftrightarrow \sigma[X]) \wedge (y_i \leftrightarrow \sigma[y_i'])$
- Use the UNSAT core to construct β repair formula

  With $X \leftrightarrow \sigma[X]$

Before repair	Repair	After repair
$\psi_i(X) \mapsto 0$	$\psi_i(X) \leftarrow \psi_i(X) \vee \beta$	$\psi_i(X) \mapsto 1$
$\psi_i(X) \mapsto 1$	$\psi_i(X) \leftarrow \psi_i(X) \wedge \neg\beta$	$\psi_i(X) \mapsto 0$

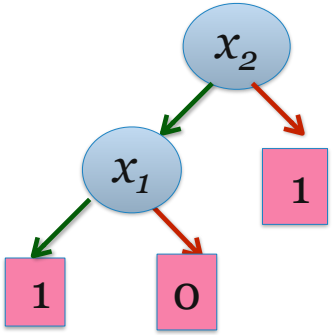
Manthan

$F(X, Y)$
$X = \{x_1, x_2\}, Y = \{y_1\}$

Data Generation

x_1	x_2	y_1
0	0	1
0	1	1
1	0	0
1	1	1

Learn Candidates Ψ



$\psi_1(x_1, x_2) = x_2 \vee (\neg x_1 \wedge \neg x_2)$

Verify Candidates

Check Satisfiability of $E(X, Y, Y')$

SAT

Find UNSAT Core of $G(X, Y)$

Construct β

Repair Candidates

Repair Cycle

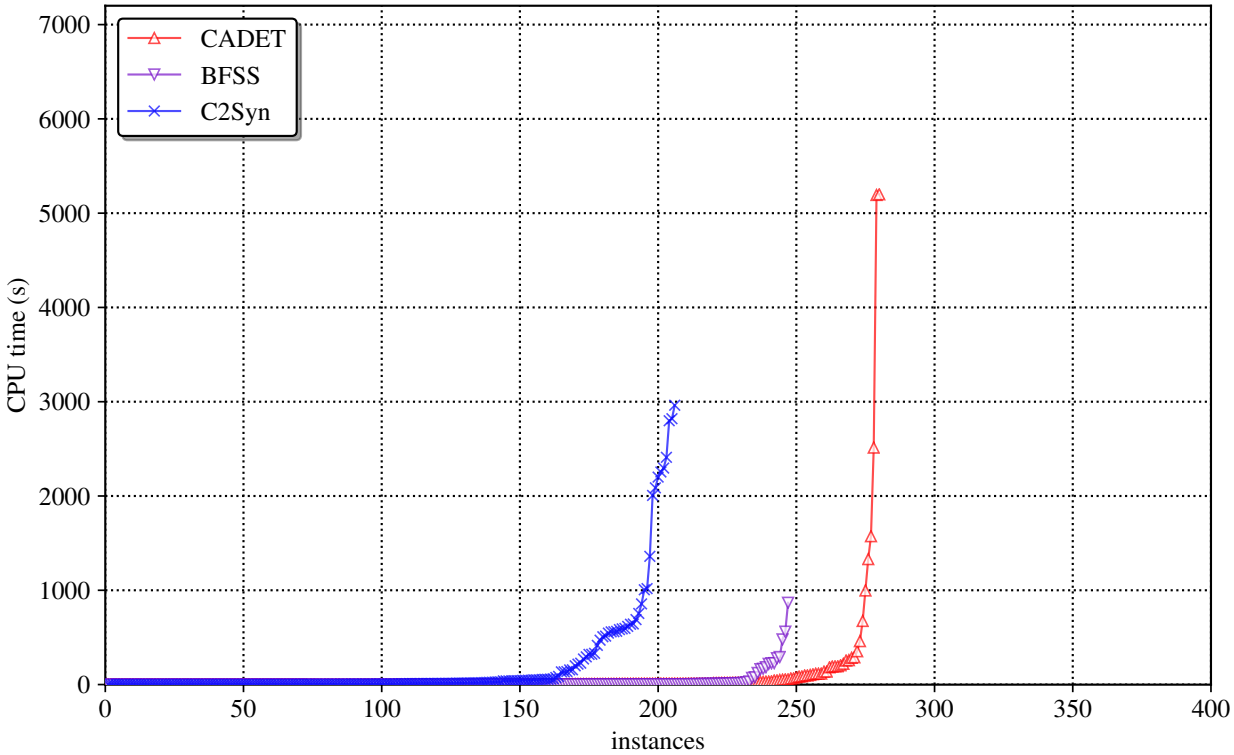
UNSAT

Return Ψ

Experimental Evaluations

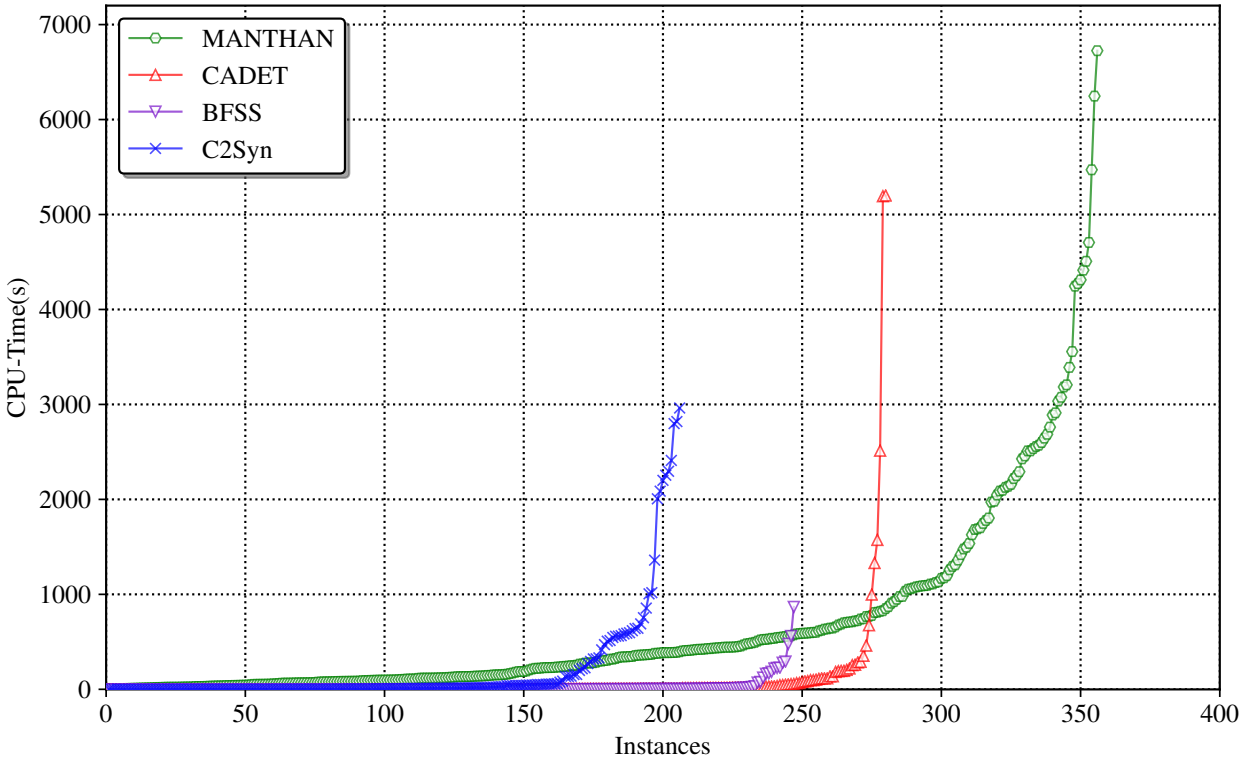
- 609 Benchmarks from:
  - QBFEval competition 2-QBF track
  - Arithmetic set (Fried, Tabajara, Vardi, 2016)
  - Disjunctive decomposition set (Akshay et al., 2017)
  - Factorization set (Akshay et al., 2017)
- Compared Manthan with State-of-the-art tools: CADET (Rabe et. al, 2019), BFSS (Akshay et al. ,2018), C2Syn (Chakraborty et al., 2019).
- Timeout: 7200s

Experimental Evaluations



C2Syn	BFSS	CADET
206	247	280

Experimental Evaluations



An increase of 76 benchmarks

Manthan \ All tools: 60

C2Syn	BFSS	CADET	Manthan
206	247	280	**356**

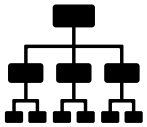
Future work: interesting questions

- What is the ideal distribution to generate the data?
- How good are the candidate functions generated by data?
- From Abstraction to Approximations in Verification?
- Can similar approach be used for program synthesis, program repair ?

Conclusion

Manthan: A Data-Driven Approach for Boolean Functional Synthesis

Constrained Sampling



Decision Tree Classifier

Automated Reasoning

Solves 356 benchmarks — state of the art could solve 280.

Opens up several interesting directions

https://github.com/meelgroup/manthan

Thanks !