Explaining the Unsolvability of Planning Problems in Hybrid Systems.

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A Planning Problem

Definition

A **Planning Problem** for hybrid system is a tuple (*Dom*, *Prob*, *Depth*):

- Dom is a hybrid automaton HA.
- Prob is a tuple (*Init*, *Goal*).
- Depth defines the bound on the plan length.

- Move the consignment from the **location 7** to **location 18** traversing a maximum of **8** cells

A Planning problem

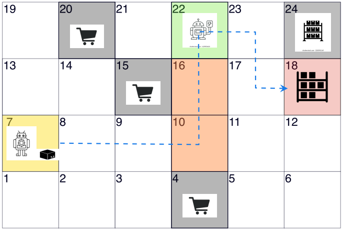
A Plan

Definition

Given a planning problem Π, a **plan** ϕ is an **action sequence** that leads an agent from *Init* to *Goal*.

Plan: right → right → right → up → up → right → down → right

Path: $\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \xrightarrow{e_{16}^{10}} l_{16} \xrightarrow{e_{22}^{16}} l_{22} \xrightarrow{e_{23}^{22}} l_{23} \xrightarrow{e_{17}^{23}} l_{17} \xrightarrow{e_{18}^{17}} l_{18} \rangle$.



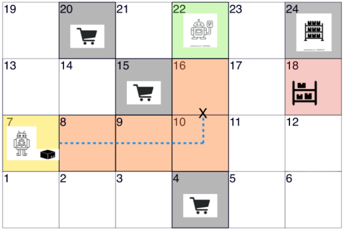
An Example of a Plan

An Unsolvable Planning Problem

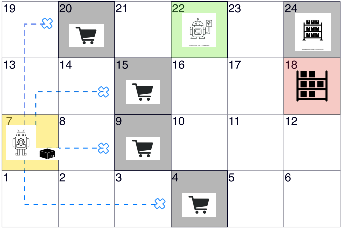
Definition

A planning problem for which **no** sequence of actions or **plan** ϕ exists that can lead from the *Init* to a *Goal* given the constraints and rules of *Dom*, is called **unsolvable**

- No plan to location 18 possible



Unsolvable (Continuous)



Unsolvable (Discrete)

Linear Constraint Encoding of a Path

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \xrightarrow{e_{16}^{10}} l_{16} \xrightarrow{e_{22}^{16}} l_{22} \xrightarrow{e_{23}^{22}} l_{23} \xrightarrow{e_{17}^{23}} l_{17} \xrightarrow{e_{18}^{17}} l_{18} \rangle$$

Location l_7 :

$t_0 \geq 0,$

$init : x_{l_7}^{in} = 0.5,\ y_{l_7}^{in} = 1.5,\ c_{l_7}^{in} = 10,$

$flow : x_{l_7}^{out} = x_{l_7}^{in} + t_0 * \dot{x},$

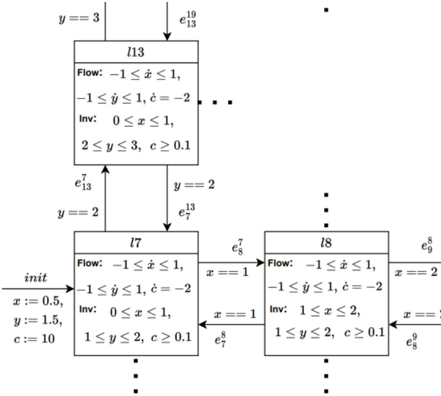
$y_{l_7}^{out} = y_{l_7}^{in} + t_0 * \dot{y},\ c_{l_7}^{out} = c_{l_7}^{in} + (-2) * t_0,$

$guard : x_{l_7}^{out} = 1,$

$invariant : 0 \leq x_{l_7}^{in} \leq 1,\ 0 \leq x_{l_7}^{out} \leq 1,$

$1 \leq y_{l_7}^{in} \leq 2,\ 1 \leq y_{l_7}^{out} \leq 2,$

$c_{l_7}^{in} \geq 0.1,\ c_{l_7}^{out} \geq 0.1;$



Hybrid Automata for Domain

Satisfiability of Path

- Thus validating the existence of a plan is **reduced to a linear program encoded** as a set of path constraints.
- **Conjunction of all the constraints** constitutes the **path formula** which we check for **satisfiability** using a SMT-based tool like z3.
- Unsatisfiability implies no valid plan exists.

We try to find exactly at which position the path becomes unsatisfiable.

Component Analysis

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \rangle, \langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \rangle, \langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \rangle \ldots$$

- Linear Program Encoding and satisfiability check for each subpath.

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \rangle : sat$$

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \rangle : sat$$

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \rangle : sat$$

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \xrightarrow{e_{16}^{10}} l_{16} \rangle : unsat$$

Relaxation

- **Relaxation:** Involves altering strict, **fixed parameters** to more flexible, **variable ranges**, i.e., loosening the constraints.
- Location 16 was not reachable from Location 10, so we start by relaxing Location 10 and move back.

Location $\mathbf{l_{10}}$(Original) :

$t_3 \geq 0,\ init : x^{in}_{l_{10}} = x^{out}_{l_9},\ y^{in}_{l_{10}} = y^{out}_{l_9},$

$c^{in}_{l_{10}} = c^{out}_{l_9}, flow : x^{out}_{l_{10}} = x^{in}_{l_{10}} + t_3 * \dot{x},$

$y^{out}_{l_{10}} = y^{in}_{l_{10}} + t_3 * \dot{y},$

$c^{out}_{l_{10}} = c^{in}_{l_{10}} + (-4) * t_3,$

$guard : y^{out}_{l_{10}} = 2,$

$invariant : 3 \leq x^{in}_{l_{10}} \leq 4,\ 3 \leq x^{out}_{l_{10}} \leq 4,$

$1 \leq y^{in}_{l_{10}} \leq 2,\ 1 \leq y^{out}_{l_{10}} \leq 2,$

$c^{in}_{l_{10}} \geq 0.1,\ c^{out}_{l_{10}} \geq 0.1;$

Location $\mathbf{l_{10}}$(Relaxed) :

$t_3 \geq 0,\ init : x^{in}_{l_{10}} = x^{out}_{l_9},\ y^{in}_{l_{10}} = y^{out}_{l_9},$

$c^{in}_{l_{10}} = c^{out}_{l_9},\ flow : x^{out}_{l_{10}} = x^{in}_{l_{10}} + t_3 * \dot{x},$

$y^{out}_{l_{10}} = y^{in}_{l_{10}} + t_3 * \dot{y},\ b \leq 0$

$c^{out}_{l_{10}} = c^{in}_{l_{10}} + (\dot{c}) * t_3,\ a \leq \dot{c} \leq b$

$guard : y^{out}_{l_{10}} = 2,$

$invariant : 3 \leq x^{in}_{l_{10}} \leq 4,\ 3 \leq x^{out}_{l_{10}} \leq 4,$

$1 \leq y^{in}_{l_{10}} \leq 2,\ 1 \leq y^{out}_{l_{10}} \leq 2,$

$c^{in}_{l_{10}} \geq 0.1,\ c^{out}_{l_{10}} \geq 0.1, ;$

Backtracking

$$\langle \xrightarrow{init} l_7 \xrightarrow{e_8^7} l_8 \xrightarrow{e_9^8} l_9 \xrightarrow{e_{10}^9} l_{10} \xrightarrow{e_{16}^{10}} l_{16} \xrightarrow{e_{22}^{16}} l_{22} \xrightarrow{e_{23}^{22}} l_{23} \xrightarrow{e_{17}^{23}} l_{17} \xrightarrow{e_{18}^{17}} l_{18} \rangle$$

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$$\vdots$$

- After each relaxation, the path goes through a satisfiability check with the modified location.
- We backtrack and relax the previous locations.
- We continue until the path becomes satisfiable.

Explanation

Figure: A Generated Explanation.

- If a path becomes feasible after relaxing certain locations, those locations are the explanation.
- The charge constraints at these locations were too strict, leading to charge depletion that blocked the path.

Results

Warehouse	# Paths	Explanation Found?	# Paths Relaxed	Path Enumeration Time (sec)	Total Time (sec)
Prob1(5,4)	192	Yes	128	0.92	20.3
Prob2(5,4)	472	Yes	31	0.84	85.7
Prob3(5,4)	15	No	15	0.53	158.2
Prob3(5,4)	542	Yes	193	7.8	1040.5
Prob3(6,4)	858	No	152	13.35	Timeout

Table: Planning Problem in warehouse instances for a plan depth of 7

Thank you!