ApproxASP – A Scalable Approximate Answer Set Counter[a]

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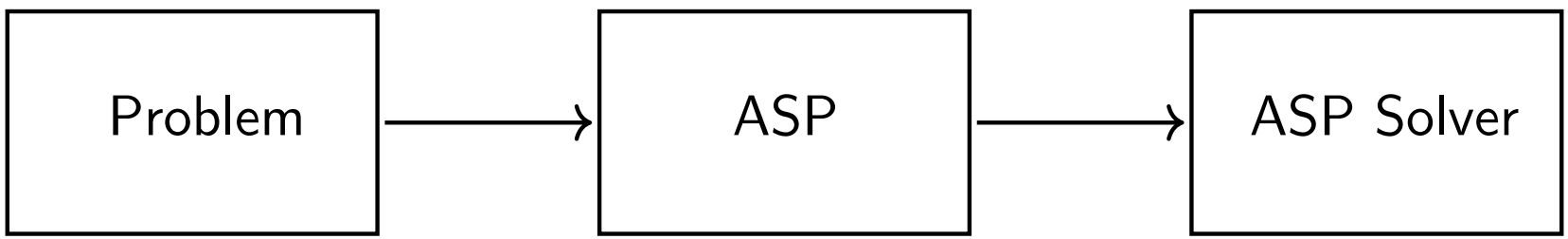
Answer Set Programming (ASP)

- ASP is a form of declarative programming:
  - logic programming
  - non-monotonic reasoning



Solution of problem $\equiv$ answer set of ASP

ASP Encoding

s-t Graph Reachability Problem

Problem Definition: Decides two nodes s and t are connected

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reached(Y) :- reached(X), edge(X,Y).
% Node t must be reached
:- ~reached(t).
```

ASP: Syntax

- A *rule-based language* for problem encoding:

$$r : h_1 \vee \ldots \vee h_\ell \leftarrow b_1, \ldots, b_k, \sim b_{k+1}, \ldots, \sim b_{k+m}$$

$$Head(r) \leftarrow Body^+(r), \sim Body^-(r)$$

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- A program $P \equiv$ *set of rules*.
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- The model of P is *answer set* or AS(P).
- Answer set solver enumerates AS(P) (ASP solving).

Answer Set Programming (ASP) (contd.)

GL-reduct

Given a program P and a set M of atoms

$$P^M = \{Head(r) \leftarrow Body^+(r) | r \in P, M \cap Body^-(r) = \emptyset\}$$

Answer Set

A set of atoms M is answer set of program P iff

- M is a model of P
- M is a (subset) minimal model of P^M

Motivation

Applications of ASP Counting

- Network Reliability (Aziz et al., 2015)
- Probabilistic Inference (Lee, Talsania, and Wang, 2017)
- Planning (Alrabbaa, Rudolph, and Schweizer, 2018)
- Navigation (Fichte, Gaggl, and Rusovac, 2022)

Challenges of Counting

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Solution: Approximate Answer Set Counting

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Approximate Counting Technique

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Experiments

$(\epsilon, \delta)-$Approximate Counting

Input

An instance I, tolerance ϵ and confidence δ

Output

estimate cnt such that

$$\Pr\left[\frac{|\,\mathsf{AS}(I)|}{(1+\epsilon)} \leq \mathsf{cnt} \leq (1+\epsilon)\cdot|\,\mathsf{AS}(I)|\right] \geq 1-\delta.$$

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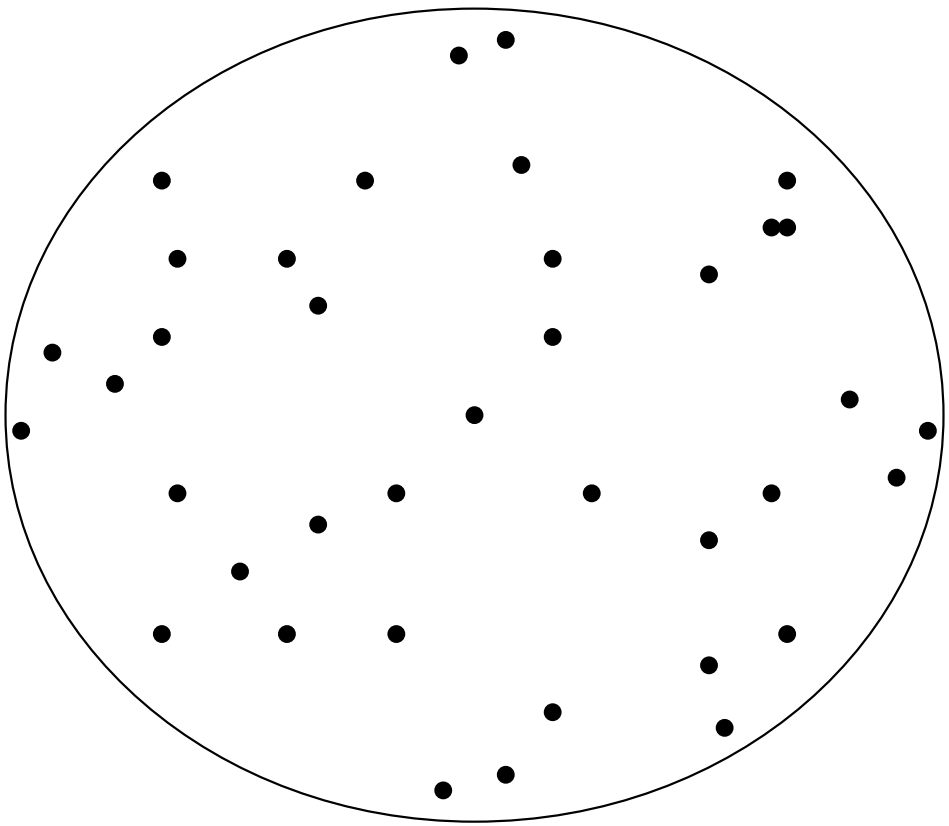
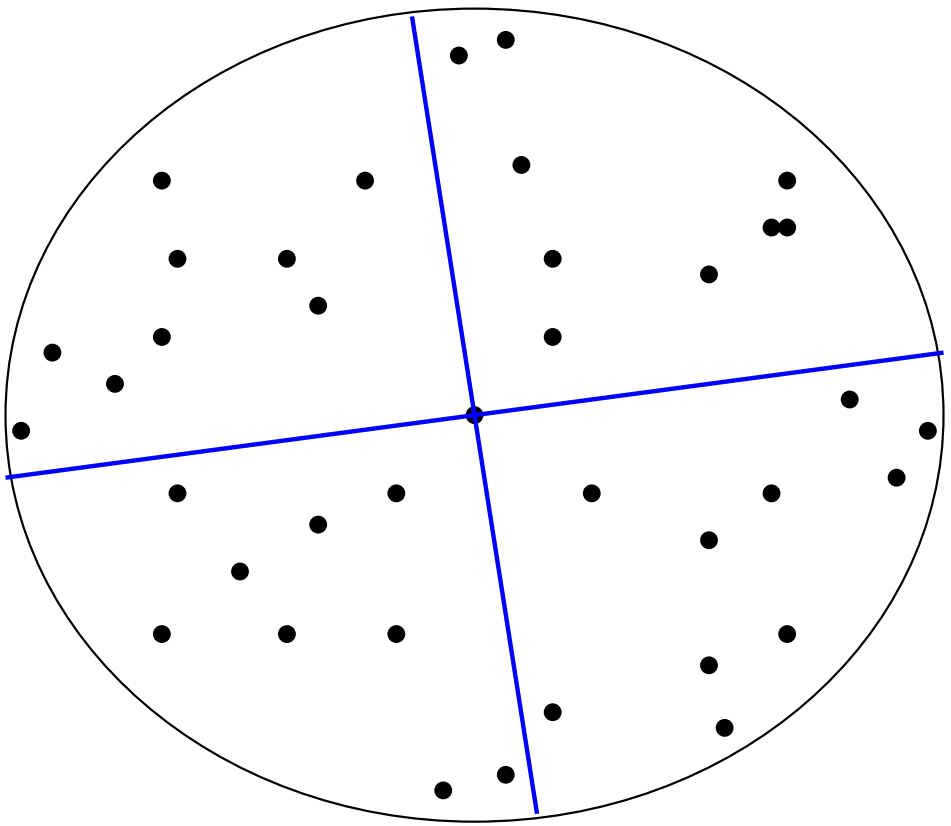
Approximate Counting

Basic Idea



Approximate Counting

Basic Idea (Contd.)



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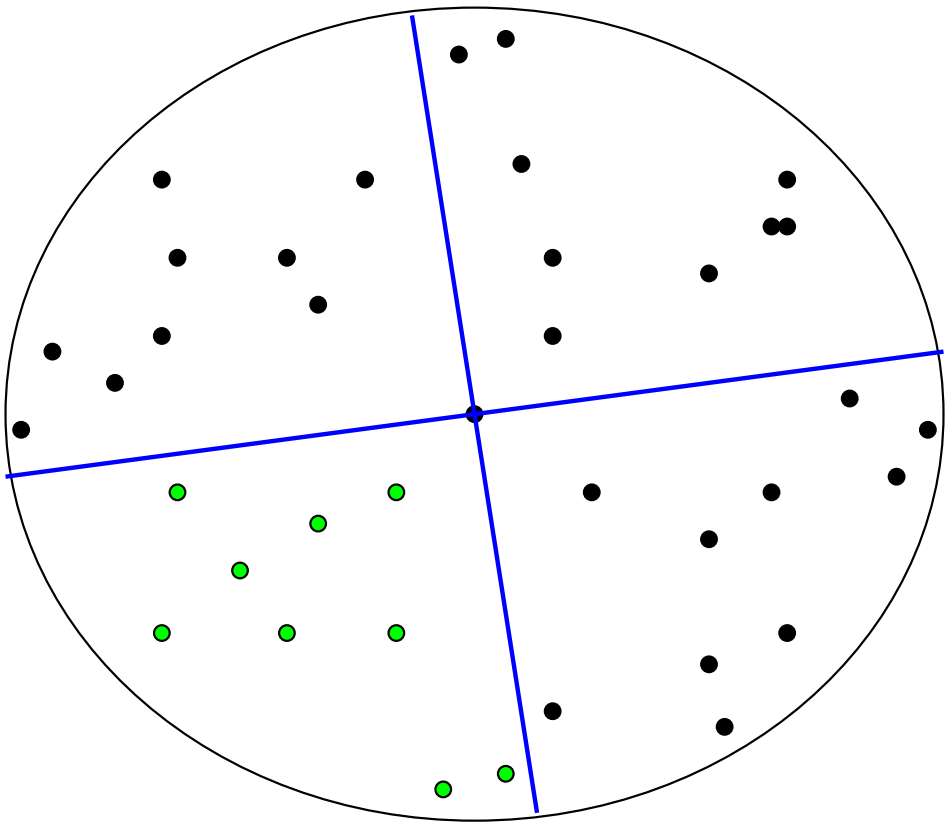

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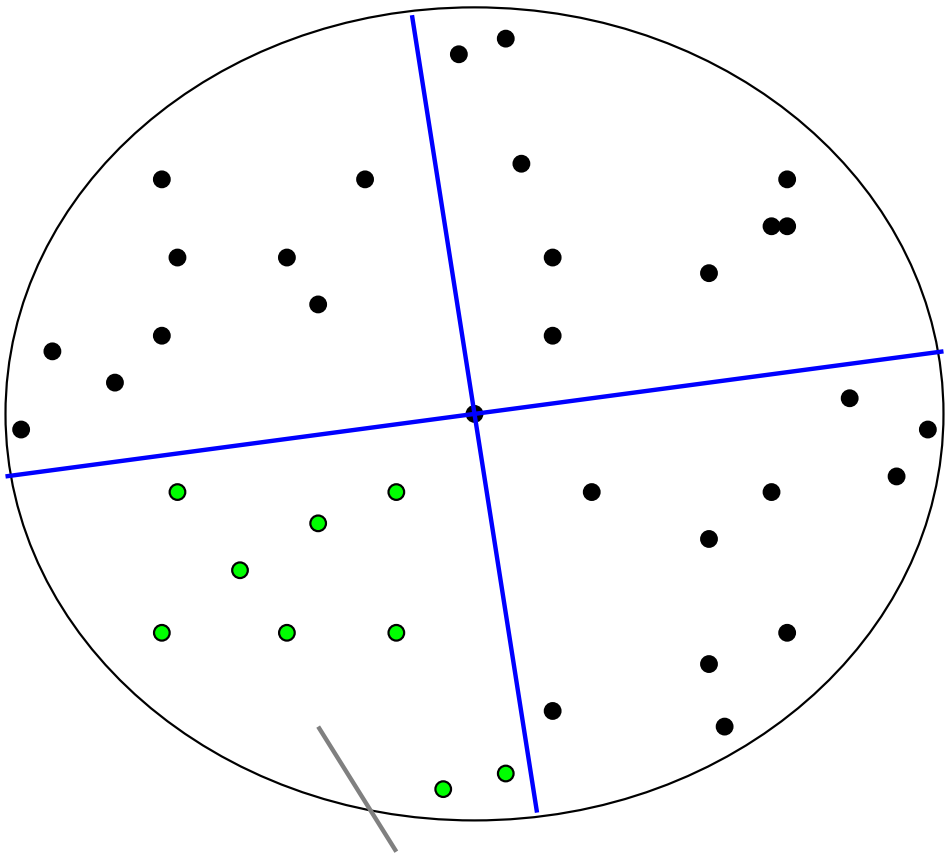

$$\boxed{\text{AppxCount} = \text{nModels} \times \text{nCells}}$$

Approximate Counting

High-level Overview

- Target: *Partition the search space uniformly and independently* (Stockmeyer, 1983) — *universal hash function* (Carter and Wegman, 1977) — Computationally Hard.

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- 2-universal hash function: *random XOR constraints* where each variable belongs to each XOR with probability $\frac{1}{2}$.
- Implementation: ASP + XOR.

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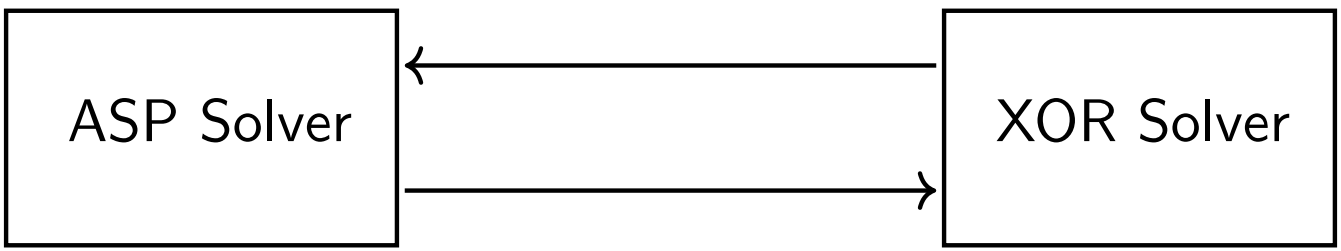
Architecture



- **ASP Solver**: Clingo (Gebser et al., 2007)
- **XOR Solver**: Han-Jiang's Gauss Jordan Elimination (GJE) (Han and Jiang, 2012)

Implementation Details

Optimizations in XOR Solving

- **Observation:** Longer XOR constraint makes the XOR solving harder (Soos, Gocht, and Meel, 2020)
- **Heuristic:** Constructing XOR constraints from *independent support* (IS)

Implementation Details

Optimizations in XOR Solving

- **Observation:** Longer XOR constraint makes the XOR solving harder (Soos, Gocht, and Meel, 2020)
- **Heuristic:** Constructing XOR constraints from *independent support* (IS)
- **Definition:** A set of atoms that uniquely defines the rest of atoms in every satisfying assignment.
- $P = \{a \vee \sim a.\ c \leftarrow \sim a.\ b \leftarrow a.\}$.
  $\{a\}$ is an independent support.

Implementation Details

IS for ASP

- We translate the ASP program to *convenient* (reasonable size) SAT program and employ off-the-self independent support calculator.

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IS for ASP

- We translate the ASP program to *convenient* (reasonable size) SAT program and employ off-the-self independent support calculator.
- For normal problem, we rely on standard translation from ASP to SAT.
- For disjunctive problem, we translate to a SAT program F such that $\forall \tau \in \mathsf{AS}(P) \rightarrow \tau \in \mathsf{Sol}(F)$, where P is the disjunctive program.

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Experiments

Instances Description

Problem Class	Encoding Name	#Instances	Source
Normal	Vertex cover	1500	Randomly generated instances
	Independent set		
	Dominating set		
	Graph Reachability		
	r-arborescence		
Disjunctive	2QBF	200	

Experiments

Baseline & Parameters

Baseline

- Clingo
- DynASP
- ApproxMC
- Ganak

Parameters

- $\epsilon = 0.8$ and $\delta = 0.2$
- Timeout $=$ 5000 sec

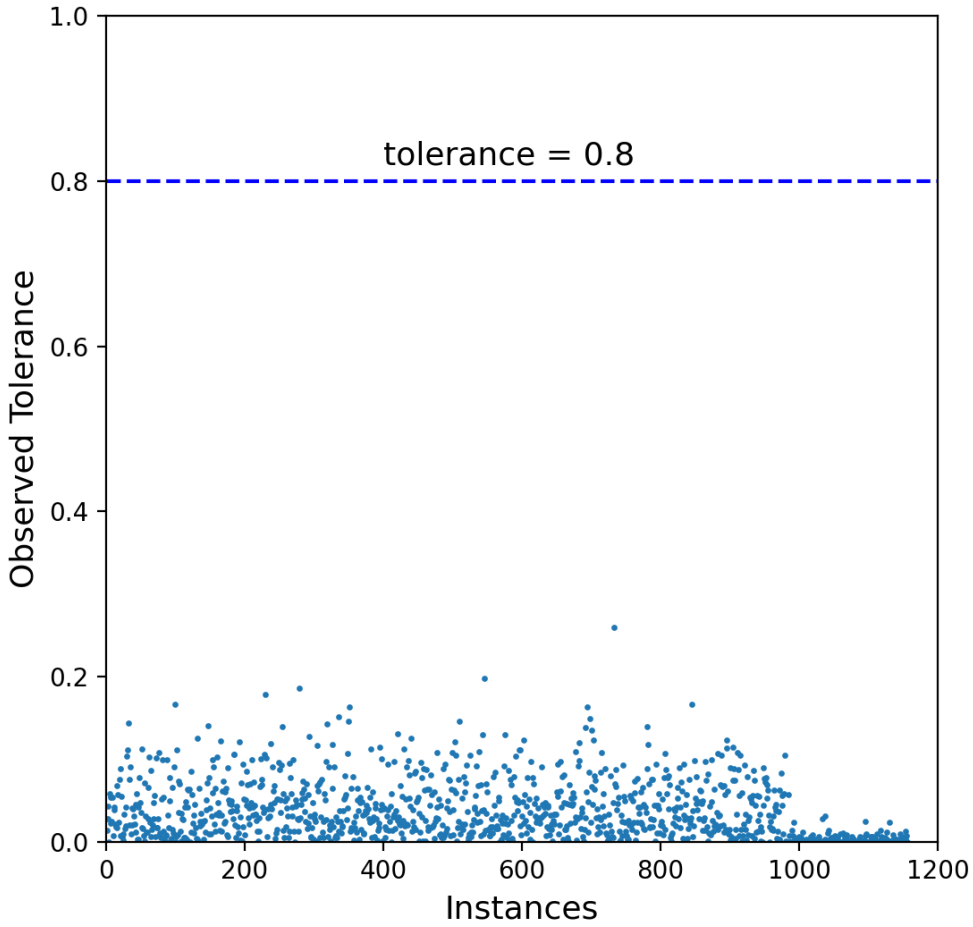
Experiments

Results

		Clingo	DynASP	Ganak	ApproxMC	ApproxASP
Normal	#Solved	738	47	973	**1325**	1323
	PAR-2	5172	9705	3606	**1200**	1218
Disjun.	#Solved	177	0	0	0	**185**
	PAR-2	1372	T	T	T	**795**

Experiments

Approximation Quality



$$\text{Observed tolerance} = \max\left(\frac{|\,\text{AS}(I)|}{\text{cnt}}, \frac{\text{cnt}}{|\,\text{AS}(I)|}\right) - 1$$

Conclusion

- We present ApproxASP – the first approximate answer set counter.
- ApproxASP solves instances that lie beyond the capability of off-the-self counting tools.

ApproxASP code

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